On the Estimation of Varying β Parameter in the Capital Asset Pricing Model (CAPM)

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Abstract

On the estimation of the Capital Asset Pricing Model (CAPM), β is assumed fixed. In the real world, β varies in accordance to changing relevant variables. This paper attempts to estimate varying β consistent with the changing relevant variables. The CAMP is applied to the energy sectoral index, PTT Public Company Limited and PTT Exploration and Production Public Company Limited stocks to estimate the varying β parameter. The variable included in the model to capture the varying β parameter is the growth rate of crude oil price. The empirical result of this study found that the traditional fixed $\beta' s$ model is recommended to use for the valuation of stocks.

Keywords: Capital Asset Pricing Model (CAPM), β parameter

Introduction

In the CAPM (capital asset pricing model), β which is the risk of the associated stock return, is estimated. The β for stock valuation is traditionally assumed fixed.

From using a large number of samples of which each sample of 250 daily observations was used to estimate β's. Then they were plotted as shown in Figure 1. It appeared that the β varied from time to time. It was unlikely that the variations of β's were due to randomness because the mean values of β's were not constant.

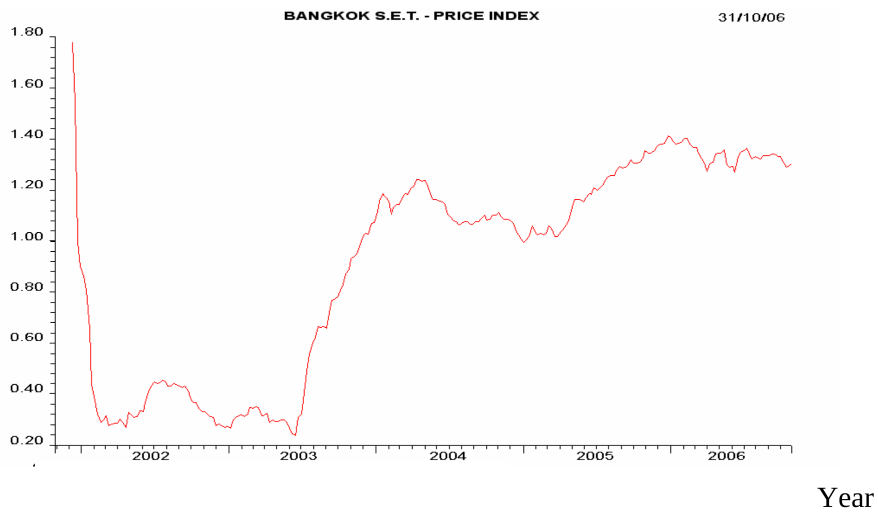

Figure 1: β of the return of PTT stock in the Stock Exchange of Thailand

Source: DATASTREAM.

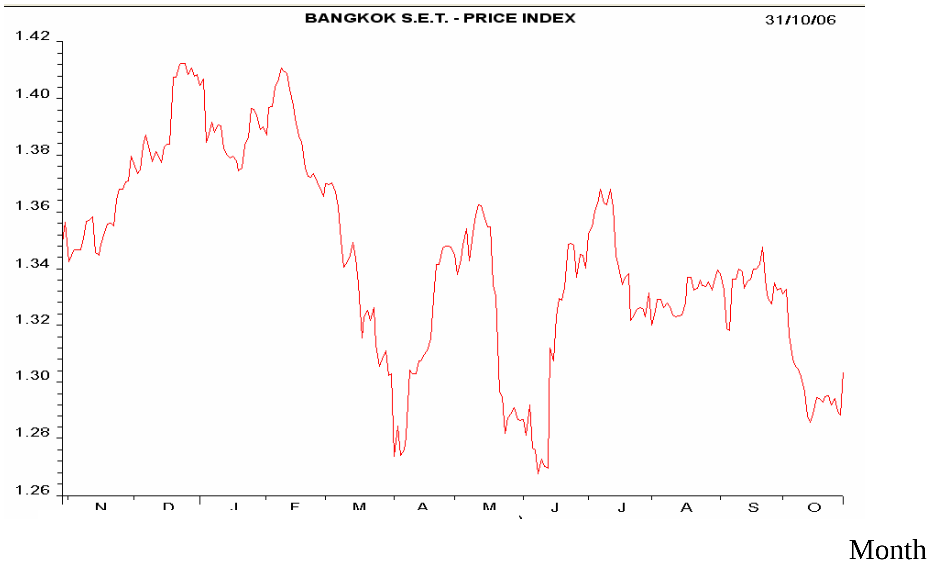

Figure 2: β of the return of PTT stock in the Stock Exchange of Thailand from the last 12 months (November 2005 – October 2006)

Source: DATASTREAM.

In the rapidly changing world economy, one might argue that the β might be latent and can be explained by a set of explanatory variables and white noise. If the β is explained significantly by significantly changing variables, stock portfolio strategists could adjust their portfolios suitably. For example, (Figure3), if $\beta = 1$, the return for the i th stock, R_i', is undervalued. This stock is recommended to "buy".

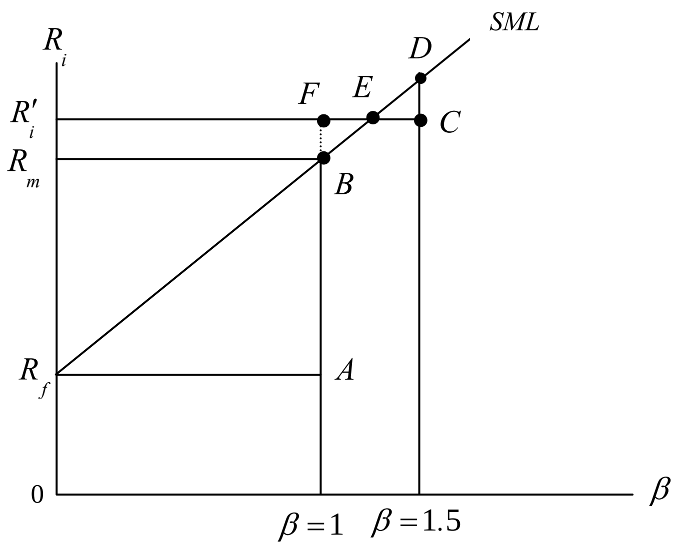

Figure 3: Security Market Line

Note: R_f = risk free rate

R_m = the rate of return from the stock market

R_i = the rate of return of the single stock under consideration

β = the parameter of the risk of the associated stock return

However, at the same rate of return R_i', suppose some economic factors affected the β, such that β was increased, e.g., changing from 1.0 to 1.5. Now it is clear that the same return is below the security market line (SML). The recommendation must be to "sell" because this stock is overvalued. Hence, the main objective of this paper is to investigate whether the β is variable, as a function of changing explanatory variables.

Methodology

Traditionally, to estimate β in the CAPM, the model is specified as follows:

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t \qquad [1]$$

where
R_t = return of a single stock
R_{mt} = disturbance
α, β = parameters

and $\varepsilon_t \sim iid\left(0, \sigma_t^2\right)$, $E\varepsilon_t\varepsilon_{t+s} = 0$ for $s \neq 0$, β is the risk of the return of the stock.
In model (1), β is assumed fixed.

However, β could be hypothesized as a varying parameter which is a function of changing explanatory variables and error term as follows:

$$\beta = \gamma_0 + \gamma_1 x_{1t} + \ldots + \gamma_k x_{kt} + v_t$$

where
$\gamma_0, \gamma_1, \ldots, \gamma_k$ = parameters
$x_{1t}, \ldots, x_{kt}$ = explanatory variables
v_t = disturbance term

$v_t \sim iid\left(0, \sigma_v^2\right)$, $Ev_t v_{t+s} = 0$ for $s \neq 0$ (Harvey, 1989; Hamilton, 1994a; Hamilton, 1994b)

In this paper, the risks of the energy sectoral index, PTT, and PTTEP stocks in the Stock Exchange of Thailand (SET) were investigated. The empirical models were specified as follows:

$$RSETENERG_t = \alpha_s + \left(\gamma_{0s} + \gamma_{1s} GOIL_t + v_{st}\right) Rm_t + \varepsilon_{st} \qquad [2]$$

$$RPTT_t = \alpha_{PTT} + \left(\gamma_{0PTT} + \gamma_{1PTT} GOIL_t + v_{PTTt}\right) Rm_t + \varepsilon_{PTTt} \qquad [3]$$

$$RPTTEP = \alpha_{PP} + \left(\gamma_{0PP} + \gamma_{1PP} GOIL_t + v_{ppt}\right) Rm_t + \varepsilon_{ppt} \qquad [4]$$

where
GOIL = growth rate of crude oil price
RSETENERG = the return of the energy sectoral index
RPTT = the stock return of PTT public company limited
RPTTEP = the stock return of PTT exploration and production public company limited

Data and empirical results

Data used in the estimation were obtained from daily data of the returns of PTT, PTTEP, energy sectoral index and the growth rate of the crude oil price from Reuters December 7, 2001 to October 30, 2006.

However, since this study used time series data, a unit root test for the stationarity of the data must be conducted. The augmented Dickey-Fuller unit root test was employed.

The results of the unit root test for stationarity of every variable showed that all variables were stationary (see Appendix).

Statistical hypotheses were set as follows:

Ho : $\gamma_{1s} = 0$ Ha : $\gamma_{1s} \neq 0$
Ho : $\gamma_{1PTT} = 0$ Ha : $\gamma_{1PTT} \neq 0$
Ho : $\gamma_{1PP} = 0$ Ha : $\gamma_{1PP} \neq 0$

The estimation of the empirical models is presented in Table 1. The empirical results indicate surprisingly that the growth rate of crude oil price has no significant effect on the β of the return of stock in any model. In the other words, we cannot reject the entire set of null hypotheses; these mean that fixed $\beta' s$ must be used.

Table 1: Estimates of Varying Parameter Models

Variable	Energy sectoral index model	PTT model	PTTEP model
		Coefficients	
C(1)	0.082466***	0.101033**	0.107998**
C(2)	0.887812***	0.960308***	0.857030***
C(3)	51751339	-22367105	-974901.7
OBVAR(1,1)	0.694579***	1.701081***	2.270459***
SSVAR(1,1)	0.255155***	0.421393***	0.461510***
Final SV1	0.000000	0.000000	0.000000
Log Likelihood	-1677.330	-2163.710	-2315.401
R-squared	0.762018	0.622739	0.526155
Durbin-Watson stat	1.928910	2.076859	2.072925

Source: Calculation Using EVIEWS.
Note: The sample included 1198 observations.
Energy sectoral index model: energy sectoral index equation of which the estimated model is
RSETENERG = C(1)+(C(2)+SV1)*RM
SV(1) = C(3)*GOIL
PTT model: PTT equation of which the estimated model is
RPTT = C(1)+(C(2)+SV1)*RM
SV(1) = C(3)*GOIL
PTTEP model: PTTEP equation of which the estimated model is
RPTTEP = C(1)+(C(2)+SV1)*RM
SV(1) = C(3)*GOIL
Significant level *** at 99 per cent level of significance
** at 95 per cent level of significance.
* at 90 per cent level of significance.

Concluding remarks

All time series variables were stationary. Conventional econometrics could be used for estimation. However, since it was observed that the β 's were not fixed, it was argued that β 's could be time variable. This study used the time varying parameter model for the analysis of β 's in the cases of the returns of the energy sectoral index, PTT and PTTEP stocks. The empirical studies showed surprising results, i.e. that the β 's were fixed in every equation specified above. This means that fixed β CAPM is recommended for stock valuations, at least in contemporary Thailand. We recommended that future research extend this methodology to other key stocks and stock exchanges around the world. However, it is suggested that the specification of β varying parameter might be revised.

References

Hamilton, J. D., 1994a. Time Series Analysis. Princeton University press.
_____.,1994b. State space Model. Chapter 50 in Robert F. Engle and Daniel L. McFadden (eds.), Handbook of Econometrics, Volume4, Amsterdam: Elsevier Science B.V.
Harvey, A. C., 1989. Forecasting. Structural Time Series Models and the Kalman Filter, Cambridge: Cambridge University press.

Appendix: Unit Root Test

Augmented Dickey-Fuller Unit Root Test on RSETENERG

ADF Test Statistic	-15.30598	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(RSETENERG)				
Sample(adjusted): 6 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RSETENERG(-1)	-0.932601	0.060931	-15.30598	0.0000
D(RSETENERG(-1))	0.007904	0.054514	0.144999	0.8847
D(RSETENERG(-2))	0.051837	0.047018	1.102495	0.2705
D(RSETENERG(-3))	-0.020348	0.039464	-0.515613	0.6062
D(RSETENERG(-4))	-0.008986	0.028967	-0.310224	0.7564
C	0.135255	0.043852	3.084322	0.0021
R-squared	0.466202	Mean dependent var		-0.000222
Durbin-Watson stat	1.999754	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.

Augmented Dickey-Fuller Unit Root Test on RPTTEP

ADF Test Statistic	-34.18628	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(RPTTEP)				
Sample(adjusted): 2 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RPTTEP(-1)	-0.989145	0.028934	-34.18628	0.0000
C	0.155665	0.057886	2.689180	0.0073
R-squared	0.494437	Mean dependent var		-0.000711
Durbin-Watson stat	1.998833	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.

Augmented Dickey-Fuller Unit Root Test on RPTT

ADF Test Statistic	-33.74938	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(RPTT)				
Sample(adjusted): 2 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RPTT(-1)	-0.976353	0.028929	-33.74938	0.0000
C	0.168111	0.055594	3.023921	0.0025
R-squared	0.488008	Mean dependent var		-0.000881
Durbin-Watson stat	1.998732	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.

Augmented Dickey-Fuller Unit Root Test on RM

ADF Test Statistic	-32.00544	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(RM)				
Sample(adjusted): 2 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RM(-1)	-0.923349	0.028850	-32.00544	0.0000
C	0.073321	0.035518	2.064311	0.0392
R-squared	0.461554	Mean dependent var		-0.000481
Durbin-Watson stat	2.001368	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.

Augmented Dickey-Fuller Unit Root Test on GOIL

ADF Test Statistic	-38.44058	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(GOIL)				
Sample(adjusted): 2 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
GOIL(-1)	-1.104891	0.028743	-38.44058	0.0000
C	0.132519	0.068586	1.932142	0.0536
R-squared	0.552883	Mean dependent var		-0.004652
Durbin-Watson stat	1.969753	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.

Augmented Dickey-Fuller Unit Root Test on RMGOIL

ADF Test Statistic	-34.42684	1% Critical Value*		-3.4386
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(GOIL)				
Sample(adjusted): 2 1198				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
GOIL(-1)	-0.995830	0.028926	-34.42684	0.0000
C	0.219207	0.094605	2.317084	0.0207
R-squared	0.497943	Mean dependent var		0.002596
Durbin-Watson stat	1.957199	Prob(F-statistic)		0.000000

Source: calculation using EVIEWS.